Nonlinear analysis of partially connected composite beams using interface elements

João Batista M. Sousa Jr.*, Amilton R. da Silva

Department of Civil Engineering, Escola de Minas, Universidade Federal de Ouro Preto, 35400-000 Ouro Preto-MG, Brazil

Received 15 July 2006; received in revised form 15 February 2007; accepted 8 July 2007
Available online 27 July 2007

Abstract

Steel–concrete beams with partial shear connection are widely used in composite construction, and their numerical simulation has received much attention in recent years. Most of the research, however, has been focused on the development of specific finite elements with either displacement, force-based or mixed formulations. This paper presents an alternative procedure for nonlinear numerical analysis of composite beams, where the partial connection between the elements is dealt with especially designed interface elements. The element developed is an extension of a classic interface element where the relative displacements are independently interpolated on the top and bottom element edges. The upper and lower composite beam components may have generic cross-sections allowing not just the classical steel-concrete composite beam, but also other assemblages of partially connected components.

Keywords: Composite beams; Partial shear interaction; Interlayer slip; Interface elements

1. Introduction

Composite beams are usually made of a steel girder which is linked to a concrete slab, with or without decking, by means of shear connectors. The combined action of steel and concrete allows the two materials, in the sagging moment region, to work where they are most effective: the compressive stresses tend to be resisted by the concrete on the top and the tensile stresses by the steel. Additionally, the bond between the elements helps to prevent local buckling of the top flange and also lateral buckling of the steel girder.

This combined action relies on the presence of shear connectors, which greatly influence the behavior of the composite beam. This influence is represented by the deformability and ductility of the connection. Economical and practical implications may lead to designs where the deformability of the connection allows the upper and lower elements to have relative axial displacements. This relative movement is often called interlayer slip in the literature.

The solution of the beam with deformable shear connection has received attention for a long time. The first solutions to the problem aimed at developing analytical models, such as the well-known Newmark differential equation, developed more than 50 years ago [1]. Recent works have employed analytical solutions to develop formulations to be used in more general cases, such as Faella et al. [2] and Ranzi et al. [3].

The difficulties of finding analytical solutions for general nonlinear cases as well as the need to consider partial interaction behavior in practical design situations led to the development of numerical solutions, mostly based on the finite element method (FEM).

In that sense, one may think of various options of dealing with the problem. Three-dimensional (3D) analysis with solid elements is obviously the strategy which will produce the most accurate results and be able to simulate the complex phenomena. Nonetheless, when dealing with practical engineering problems, 3D analysis often becomes too expensive and sometimes unavailable. Issues such as mesh generation may become cumbersome. Beam-column finite elements, on the other hand, may be dramatically cheaper in computational terms and, to engineering purposes, correctly model composite beams well into...
the non-linear range. Therefore numerical solutions for beams with deformable shear connection are still an active field of research.

A brief review of recent work on partially connected composite beams follows. Wang [4] proposed analytical solutions to evaluate deflections on composite beams and compared the results with experimental and numerical linear results. Owen et al. [5] also employed a semianalytical model in the development of specialized finite elements to simulate partial connection behaviour, in a displacement-based formulation. Dall’Asta and Zona [6,7] developed a family of displacement-based elements for the nonlinear analysis of partially connected beams. Later they extended their work with the development of three-field mixed elements [8] and discussed the relative merits of the formulations [9]. Mixed or force-based elements were also the subject of works by Salari and Filippou [11], Salari and Spacone [12] and Ayoub [13]. Roughly speaking, the advantage of the displacement-based formulations are the simplicity and ease of implementation. In some cases, however, they may display locking phenomena [7]. Force-based elements, on the other hand, display more precise results with fewer elements per member, satisfy equilibrium equations pointwise, but their formulation and implementation is not simple due to the state determination procedures.

The purpose of this paper is to present an alternative numerical solution for the analysis of composite beams with deformable shear connection. This solution involves the development of a specific zero-thickness interface element to represent the behaviour of the connection, associated with inelastic two-noded beam elements. The upper and lower parts of the composite beam may be of generic cross-section, and analytical integration of section forces and tangent moduli are employed, thus endering a powerful and robust numerical tool for the analysis of beams with partial interaction.

2. Kinematical model

The usual kinematical hypotheses underlying partially connected beams have been established for a long time and may be summarized as follows: (i) plane sections of the upper and lower parts remain plane, (ii) the vertical displacement is the same for both components, as well as its derivatives, so that the curvature is unique and no uplift takes place, (iii) the displacements and rotations are small so that the section rotation angle is equal to the first derivative of the vertical deflection (Euler-Bernouilli assumption). Assuming that the composite beam lies in the xy plane, the displacement field is given by (Fig. 1)

\[ u_x(x, y) = u^0_x(x) + (y - y_2)\theta(x), \]
\[ v_y(x, y) = v^0_y(x), \]

where \( x = 1 \ldots 2 \) represents the upper and the lower element, respectively, and \( y_2 \) is the reference axis for each constituent element, which is usually taken at the centroid.

From (1) and (2) the strains may be obtained, for the small-displacement case, as

\[ \varepsilon_{xx} = \frac{\partial u^0_x}{\partial x} - y\frac{\partial \theta}{\partial x} = \frac{\partial u^0_x}{\partial x} - y\frac{\partial^2 v}{\partial x^2}. \]

From these kinematical relations most of the FE formulations are derived for the simulation of partially connected composite beams.

3. Interface elements

The finite element solution to problems in which adjacent elements have different tangential displacements has been investigated by engineers and researchers for decades. Interface elements must be able to predict and allow slip and debonding between two bodies in contact, or separated by a thin material layer. Constitutive relations may include slip, non-slip, separation and rebonding. In some cases, special formulations and contact detection algorithms must be employed.

The simulation of an interface zone can be performed, according to Karabatakis and Hatzigogos [14], by means of (a) thin continuum finite elements, (b) linkage elements in which opposite nodes are connected by discrete springs or (c) interface elements with zero or finite thickness.

Thin continuum finite elements have been employed for geotechnical problems. Their main advantage is that no new element formulation is introduced, but they present numerical ill-conditioning for low thickness/length ratios [15].

Linkage elements for interface problems were first employed by Herrmann [16]. Their formulation is quite simple as springs are attached to the element nodes in order to simulate relative displacements. Some formulations for the analysis of composite beams employ this kind of strategy. Gattesco [17] developed a numerical solution for the nonlinear analysis of composite beam...
beams by attaching springs to the element ends and with fibre integration of cross-sectional properties. More recently, this type of approach was employed by Ranzi et al. [18] and Gara et al. [19] on the simulation of composite beams with longitudinal as well as transverse partial interaction.

The GTB element developed by Goodman et al. [20] was the first zero-thickness interface element. It enables the simulation of relative displacements within two adjacent finite elements, by the independent interpolation of the relative displacements on opposite edges. The strains are then evaluated from these relative displacements, and from the expression for the strain energy the element stiffness matrix may be obtained. The stiffness expression contains values of the element height and for the linkage (spring) based elements.

Extensions of the original GTB element were later proposed by the independent interpolation of the relative displacements within two adjacent finite elements, and it was pointed out that they apparently have not yet been considered for the simulation of partial connection of composite steel–concrete beams.

Despite all the applications carried out in the fields of solid and geomechanics, interface elements derived from the GTB element apparently have not yet been considered for the simulation of partial connection of composite steel–concrete beams, which is a typical situation where relative displacements between adjacent elements take place.

The interface element formulation proposed here stems from the original GTB element. As for the kinematical inconsistency, no special care was taken as, with the interface elements combined in a mesh, this abnormal behaviour did not show up in any of the composite beam examples tested.

The interface element to be used for the simulation of the deformable connection has two translational and one rotational displacement at each node, Fig. 2. The displacement field has for the tangential relative displacement

\[ w_h(x) = u_2^0(x) - u_1^0(x) + (y_2 - d) \theta_2(x) - (y_1 - d) \theta_1(x) \]  

(4)

and for the normal relative displacement

\[ w_v(x) = v_2(x) - v_1(x). \]  

(5)

For steel–concrete composite beams, the relative vertical displacement is often neglected. The nonlinear force–displacement relationships for the tangential and normal relative displacements are

\[ S_h = S_h(w_h) \quad \text{and} \quad N_h = N_h(w_v). \]  

(6)

Application of the principle of virtual work to the isolated interface element leads to

\[ \delta W = \int (S_h \delta w_h + N_h \delta w_v) \, dx. \]  

(7)

The variation of the displacement \( w_h \) is given by

\[ \delta w_h(x) = \delta u_2^0(x) - \delta u_1^0(x) + (y_2 - d) \delta \theta_2(x) - (y_1 - d) \delta \theta_1(x). \]  

(8)

From Eq. (8) it becomes clear that one could generate an interface element associated with Timoshenko beam elements. In the present work, however, focus is on reinforced concrete–steel composite beams, for which the Kirchhoff assumption is normally considered. Assuming that the displacement interpolation in the interface element is aimed at its compatibility with cubic Kirchhoff-type elements leads to

\[ \delta w_h(x) = \delta u_2^0(x) - \delta u_1^0(x) + (y_2 - d) \delta v_2(x) - (y_1 - d) \delta v_1(x). \]  

(9)

For the vertical displacement

\[ \delta w_v(x) = \delta v_2(x) - \delta v_1(x), \]  

(10)

so that the virtual work expression reads

\[ \delta W = \int_0^L \left( S_h \delta u_2^0(x) - S_h \delta u_1^0(x) + (y_2 - d) \delta v_2(x) - (y_1 - d) \delta v_1(x) \right) \, dx \]  

(11)

The element degrees of \( q \) freedom are (Fig. 2)

\[ q^T = \begin{bmatrix} q_{uh}^T & q_{vh}^T & q_{ut}^T & q_{vt}^T \end{bmatrix}, \]  

(12)

where one has

\[ q_{ub} = \begin{bmatrix} q_{u1} & q_{u2} \end{bmatrix}^T, \quad q_{ut} = \begin{bmatrix} q_{u3} & q_{u4} \end{bmatrix}^T, \]  

(13)

The displacement interpolation for the element is written in matrix form as

\[ u = Q q = \begin{bmatrix} u_1^0 & u_2^0 & v_1 & v_2 \end{bmatrix} = \begin{bmatrix} \phi_u^T & 0 & 0 & 0 \\ 0 & \phi_u^T & 0 & 0 \\ 0 & 0 & -\phi_v^T & \phi_v^T \end{bmatrix} \begin{bmatrix} q_{ub} \\ q_{ut} \\ q_{vh} \\ q_{vt} \end{bmatrix}. \]  

(14)

The elements of the interpolation function vectors \( \phi_u \) and \( \phi_v \) are the linear and cubic displacement interpolation functions, respectively. The variations of the displacement fields are obtained from

\[ \delta (\bullet) = \delta q^T \tilde{C}(\bullet) \]  

(15)

where \( \bullet \) is either \( u_1^0, u_2^0, v_1, v_2, \) or \( v_1 \) and \( q \) is the vector of the element degrees of freedom. From the internal virtual work expression one gets in a standard fashion the internal
force vector
\[ \mathbf{f}_{\text{int}} = \int_0^\ell \left[ S_b \left( \frac{\partial u_0^0}{\partial q} - \frac{\partial u_0^1}{\partial q} + (y_2 - d) \frac{\partial v'_1}{\partial q} - (y_1 - d) \frac{\partial v'_2}{\partial q} \right) + N_b \left( \frac{\partial v_2}{\partial q} - \frac{\partial v_1}{\partial q} \right) \right] \, dx. \]  

The derivation of \( \mathbf{f}_{\text{int}} \) with respect to the displacements gives the element tangent stiffness matrix
\[ \mathbf{K}_f = \int_0^\ell \left[ \frac{\partial u_0^0}{\partial q} + (y_2 - d) \frac{\partial v'_1}{\partial q} - (y_1 - d) \frac{\partial v'_2}{\partial q} \right] \, dx. \]  

Introducing the interpolation functions in (16) and (17), the internal force is obtained as
\[ \mathbf{f}_{\text{int}} = \int_0^\ell \left\{ \begin{array}{l} -S_b \phi_u \\ S_b \phi_u \\ (y_2 - d) S_b \phi'_u + N_b \phi_v \end{array} \right\} \, dx \]  

and for the tangent stiffness matrix
\[ \mathbf{K}_f = \int_0^\ell \left[ \begin{array}{l} \phi_u \left( \frac{\partial S_b}{\partial q} \right)^T \\ (d - y_1) \phi'_u \left( \frac{\partial S_b}{\partial q} \right)^T - \phi_v \left( \frac{\partial N_b}{\partial q} \right)^T \\ (y_2 - d) \phi'_u \left( \frac{\partial S_b}{\partial q} \right)^T + \phi_v \left( \frac{\partial N_b}{\partial q} \right)^T \end{array} \right\} \, dx. \]  

Upon substitution of the derivatives of \( S_b \) and \( N_b \) with respect to the element displacements, and defining the following symbols:
\[ \mathbf{\Phi}_{uu} = \phi_u \phi_u^T, \quad \mathbf{\Phi}_{uv} = \phi_u \phi_v^T, \quad \mathbf{\Phi}_{vv} = \phi_v \phi_v^T, \quad \mathbf{\Phi}_{vu} = \phi_v \phi_u^T, \quad \mathbf{\Phi}_{v'u'} = \phi_{v'} \phi_{u'}^T, \]  

one gets the final expression for the tangent stiffness:
\[ \mathbf{K}_f = \int_0^\ell \left\{ \begin{array}{l} S_b^T \mathbf{\Phi}_{uu} - S_b^T d_1 \mathbf{\Phi}_{uv'} - S_b^T d_2 \mathbf{\Phi}_{vv'} - S_b^T d_3 \mathbf{\Phi}_{v'u} - S_b^T d_4 \mathbf{\Phi}_{v'v'} \quad -S_b^T d_5 \mathbf{\Phi}_{uu} \quad -S_b^T d_6 \mathbf{\Phi}_{uv'} \quad -S_b^T d_7 \mathbf{\Phi}_{vv'} \quad -S_b^T d_8 \mathbf{\Phi}_{v'u} \quad -S_b^T d_9 \mathbf{\Phi}_{v'v'} \\ S_b^T d_1 \mathbf{\Phi}_{uv} + N_b^T \mathbf{\Phi}_{vv} - S_b^T d_2 \mathbf{\Phi}_{v'u} \quad S_b^T d_3 \mathbf{\Phi}_{v'v'} - N_b^T \mathbf{\Phi}_{vv} \quad S_b^T d_4 \mathbf{\Phi}_{v'v'} \quad S_b^T d_5 \mathbf{\Phi}_{v'v'} + N_b^T \mathbf{\Phi}_{vv} \end{array} \right\} \, dx. \]  

The previous expressions were developed considering different positions of the reference axes of each part of the composite beam (e.g., centroids). These reference axes need not be at any particular position. If, for instance, both axes are positioned exactly over the interface line, one gets \( y_1 = y_2 = d \) and much simpler expressions are obtained:
\[ \mathbf{K}_f = \int_0^\ell \left\{ \begin{array}{l} S_b^T \mathbf{\Phi}_{uu} \quad 0 \\ 0 \quad S_b^T \mathbf{\Phi}_{vv} \quad 0 \quad N_b^T \mathbf{\Phi}_{vv} \end{array} \right\} \, dx. \]  

It is important to note that in this new situation the relative displacements become (see Eq. (4))
\[ w_b(x) = u_b^0(x) - u_a^0(x) \]  

and the slip becomes a linear function inside each element. There are no mismatching polynomial terms coming from transverse displacement interpolation, so the spurious oscillations which appear on the slip solution are eliminated. This problem is present whenever the strain contributions come from different fields whose polynomials are unbalanced, and has been described by Dall’Asta and Zona [7]. However, a beam element with eccentric axes may have poorer quality and produce imprecise solutions [25,26].

4. Beam elements

The interface element developed in this work may be associated to a two-noded Kirchhoff beam-column element with displacement and rotational DOF. As the main concern here is the modeling of composite steel–concrete beams with interlayer slip, the elements implemented had cubic interpolation of transverse displacements and linear interpolation of axial displacements.

Material nonlinearity is considered at the integration points at the cross-section level of the beam elements. The relevant quantities to be evaluated for each beam element are the resistive forces
\[ N = \int \sigma_x \, dA_x \quad M = \int \sigma_x y \, dA \]  

where the following symbols were introduced:
\[ d_1 = d - y_1 \quad \text{and} \quad d_2 = y_2 - d, \]  

and \( S_b' \) and \( N_b' \) are the horizontal and vertical connection tangent stiffnesses. From the expression of the stiffness matrix the order of numerical integration required for the interface element may be deduced. It may be seen that due to terms such as \( \mathbf{\Phi}_{vv} \), polynomial terms up to the sixth degree arise. Therefore, exact integration is possible if, with Gaussian quadrature, four points are used. In fact, the tests conducted revealed that numerical errors occur with less than four Gauss points.
The fibre method may be very easily implemented into an FE code but presents the disadvantage of requiring specific procedures for each type of cross-section geometry. Moreover, its numerical precision depends on the number of fibres (layers) employed.

Alternatively, analytical integration of sectional properties (forces and tangent stiffness) has been investigated by a number of researchers and several strategies have been developed [28,29]. The analytical strategy employed here considers the possibility of polygonal cross-sections of various different materials in the same beam, and relies on a piecewise polynomial description of the uniaxial stress–strain relations. By means of a contouring algorithm and using Green’s theorem, the area integrals are transformed into line integrals on the cross-section boundaries, see details as presented by Sousa and coworkers [30,31]. The main advantage of this strategy is the possibility of considering generic polygonal cross-section geometries, associated to a fast and exact evaluation of the cross-section properties.

5. Numerical examples

5.1. Comparison with Newmark analytical solution

For linear elastic material models and linear force–slip relation in the connection, analytical solutions may be obtained from Newmark’s differential equation. The first example will compare the results from different discretizations of the finite element mesh with the analytical solution to validate the numerical analysis in the simplest case.

The differential equation will be presented here with the curvature \( \chi \) as dependent variable, for constant distributed load \( q \), with \( K \) the (constant) connection stiffness and \( h \) the distance between the centroids of the upper and lower components [2]:

\[
\chi'' - \chi^2 = -\frac{q}{(EI)_{\text{free}}} - \chi^2 \frac{M}{(EI)_{\text{full}}} = -\frac{q}{(EI)_{\text{free}}} - \chi^2 \frac{q(Lx - x^2)}{2(EI)_{\text{full}}},
\]

(26)

In (26), \((EI)_{\text{free}}\) is the composite section stiffness with zero shear connection:

\[
(EI)_{\text{free}} = E_c I_c + E_s I_s
\]

(27)

with \(E_c\) and \(E_s\) the elastic modulus of concrete (upper) and steel (lower) section components, and \(I_c\) and \(I_s\) their second order moments. The full composite section stiffness \((EI)_{\text{full}}\) is given by the expression

\[
(EI)_{\text{full}} = (EI)_{\text{free}} + h^2 (EA)^*.
\]

(28)

where

\[
(EA)^* = \frac{E_c A_c E_s A_s}{E_c A_c + E_s A_s}
\]

(29)

and

\[
\chi^2 = \frac{K(EI)_{\text{full}}}{(EA)^*(EI)_{\text{free}}}.
\]

(30)

The exact solution for the curvature is given by solving (26) for \( \chi \):

\[
\chi(x) = C_1 e^{2x} + C_2 e^{-2x} - Q + \frac{q x (L - x)}{2(EI)_{\text{full}}},
\]

(31)

where

\[
Q = \frac{q}{\chi^2(EI)_{\text{full}}} - \frac{q}{\chi^2(EI)_{\text{free}}}.
\]

(32)

The integration constants \(C_1\) and \(C_2\) of Eq. (31) may be found by imposing zero curvature on the ends of the simply supported beam, i.e., \( \chi = 0 \) at \( x = 0 \) and \( L \):

\[
C_1 = \frac{Q}{e^{2L} - e^{-2L}},
\]

(33)

\[
C_2 = \frac{Q}{1 - e^{-2L}} - e^{2L}.
\]

(34)

The vertical displacement is given by integrating Eq. (31) twice with respect to \( x \), since \( \chi = v'' \):

\[
v(x) = -\frac{C_1}{2} e^{-x} - \frac{C_2}{2} e^{x} + C_3 x + C_4 + \frac{Q L}{12(EI)_{\text{full}}} x^3 + \frac{q L}{24(EI)_{\text{full}}} x^4,
\]

(35)

where the new integration constants are found imposing zero vertical displacements at the ends of the beam, i.e., \( v = 0 \) at \( x = 0 \) and \( L \):

\[
C_3 = \frac{C_1}{2} e^{-x} - \frac{C_2}{2} e^{x} - \frac{QL}{2} + \frac{QL^3}{24(EI)_{\text{full}}},
\]

(36)

\[
C_4 = \frac{(C_1 + C_2)}{2}.
\]

(37)

In order to evaluate the relative slip between the components, one may depart from the bending moment equation

\[
M(x) = \chi(EI)_{\text{free}} + F h,
\]

(38)

where \(F\) is the connection force, whose derivative in this case is linearly related to the slip \((F' = Ks)\). After derivation, Eq. (38) gives

\[
M'(x) = \chi'(x)(EI)_{\text{free}} + F'h = \chi'(EI)_{\text{free}} + K hs(x).
\]

(39)

The relative slip is then given by

\[
s(x) = \frac{M'(x) - \chi'(x)(EI)_{\text{free}}}{Kh}.
\]

(40)

After substitution for \(M(x)\) and \(\chi'(x)\) one gets the expression for the slip:

\[
s(x) = \frac{(EI)_{\text{free}}(C_2 e^{-x} - C_1 e^{x}) + \frac{q h (EA)^*}{K(EI)_{\text{full}}} \left( \frac{L - x}{2} \right)}{Kh}.
\]

(41)

In order to assess the results of the proposed formulation, the example will consider a simply supported composite beam...
with 6 m span (Fig. 3), subjected to distributed load of 80 kN/m. The cross-section is composed of a steel profile with 300 mm height, 150 mm width, 10 mm web thickness and 16 mm flange thickness. The material properties are $E_c = 30,500 \text{ MPa}$ and $E_s = 200,000 \text{ MPa}$.

Three different FE meshes (2, 3 and 12 elements) and two different levels of connection stiffness ($zL = 1$ and 20) were employed in the analyses. Moreover, the analyses are carried out with the beam axes located on their centroids. Four Gauss points were used for the numerical integration of the connection stiffness, and two Gauss points for each beam element.

Fig. 4 displays the results for the weak shear connection ($zL = 1$) and it may be seen that the results converge to the analytical solution well. Fig. 5 depicts the results for the strong shear connection ($zL = 20$), where it may be seen that the results for the slip present an oscillatory behaviour which characterizes the slip locking, even for the finer mesh.

In order to check if the slip locking may be eliminated by the translation of the reference axis to the interface line,
the same problem with strong shear connection will be analysed, for the same sequence of meshes. Fig. 6 depicts the results for the strong shear connection with axis on the interface line. It may be seen that the spurious oscillation on the slip disappears with each element presenting a linear variation for the slip. The results for the displacements indicate that the effect of translation of axes (eccentricity) does not lead to significant errors in displacement evaluation. In this situation the coarse meshes provide a displacement pattern which is more flexible than the analytical solution.

5.2. Beam analysed by Ranzi et al. [32]

In a recent work, Ranzi et al. [32] presented a comparative study of different solution schemes for composite beams with partial interaction, namely finite difference and finite element methods, direct stiffness method and analytical solution. Those authors provided solutions for simply supported beams and for a propped cantilever, for two different levels of connection stiffness, controlled by the dimensionless parameter $zL$, where $z$ is given in Eq. (30).

In this example solutions from different strategies are compared with the solution with interface elements for the case of a propped cantilever with four elements. The length of the cantilever beam is 25 m and it is subjected to a uniformly distributed load of 35 kN/m. The cross-section is composed of a
rectangular concrete slab with \(2300 \times 230\) mm with 1% reinforcement, a 1200WB455 (Australian welded beam) which has both flanges 500 mm \(\times\) 40 mm and a web 1120 mm \(\times\) 16 mm. For steel the elastic modulus is taken as 210 000 MPa, and for concrete it is 34 200 MPa. The relative connection stiffness in this example was taken as \(zL = 1\) (weak connection) and 50 (strong connection).

In Figs. 7 and 8, FEM 8DOF and FEM 10DOF refer to eight and ten degrees of freedom displacement-based finite elements, INT1 refers to interface elements with axes on the beam centroids and INT2 refers to interface elements with axes translated to the interface. The exact solution refers to the solution with a very fine mesh (200 elements) with the 10DOF elements.

It may be seen that for the low value of shear connection, the interface element with centroidal axes (INT1) provides results identical to the 8DOF element, which is predictable as they have similar interpolation schemes. Translating the axes to the interface line (INT2) renders a stiffer element, with linear slip distribution and also a poorer representation of curvature (which tends to a constant value inside the element).

For the higher connection stiffness, however, the displacement pattern for the INT2 element is much closer to the exact solution than that of the 8DOF and the INT1 element, which again coincide and are stiffer. As for the slip values, the 8DOF and INT1 elements present a strong oscillatory behaviour typical of slip locking, and also a poor curvature representation.

![Fig. 8. Example 2—displacement, slip and curvature with \(zL = 50\).](image)

![Fig. 9. Example 3—continuous beam.](image)

![Fig. 10. Example 3—cross-section.](image)
5.3. Nonlinear analysis example [9]

In this example, results from the numerical analysis of a continuous composite beam with nonlinear material properties are compared with the results with the methodology proposed here. This structure has been previously analyzed by Dall’Asta and Zona [9] with displacement as well as mixed finite elements. They recognized this example as a difficult test for composite beam elements, due to the high slip gradient, strain localization, concrete cracking over the support and softening in the sagging region. Figs. 9 and 10 depict the geometry and cross section of the beam.

The material stress–strain relationships are represented in Fig. 11. For the beam and the reinforcement steel, the properties are given in Table 1, and the elastic modulus is taken as 200 000 MPa. For the concrete, the CEB–FIP Model Code relationship, with peak stress \( f_c \) of 33 MPa is employed [33]. The model for the slip connection stiffness is that of Öllgaard [34]:

\[
F = F_{\text{max}}(1 - e^{-\beta s})^\alpha \quad \text{with} \quad \alpha = 0.558 \quad \text{and} \quad \beta = 1 \text{mm}^{-1}. \quad (42)
\]

In Eq. (42), \( F_{\text{max}} \) is taken as 1240 kN/m and \( s_u \) as 6 mm. The reinforcement axis was positioned at a distance of 2.5 mm of the top and bottom slab faces.

In order to employ the analytical integration of cross-section properties approximate polynomial relationships were employed for the materials. Fig. 9 displays the continuous composite beam, whose cross-section is depicted in 10.

Three different meshes are employed in the numerical analyses, namely, with 8, 12 and 16 elements. Fig. 12 shows the load–displacement relations for these meshes along with the results from Dall’Asta and Zona [9]. In these graphs, 10DOF refers to displacement-based elements with 10DOF called PE112 in the original paper. It may be seen that for the eight element mesh the results are stiffer for the interface element, but they converge to the same values as the mesh is refined.

5.4. Comparison with a force based element

Salari and Spacone [12] developed a force-based finite element to model composite beams with deformable shear connection and compared their results with a displacement-based formulation using cubic transverse displacement interpolation functions and parabolic interpolation functions for the axial displacements. They presented the example of a two-span continuous composite beam with a concentrated load applied at each mid-span (Figs. 13 and 14).

The material uniaxial stress–strain relationships are depicted in Fig. 15. In the present work polynomial approximations were employed in order to evaluate sectional properties analytically. In this example there is a softening branch in the concrete stress–strain relationship, in both tension and compression.

Fig. 16 shows the results of the load–displacement behaviour for the continuous beam, compared with the results in [12]. Also displayed is the load–displacement curve for a 10DOF finite element (cubic transverse and quadratic axial polynomial interpolation), implemented by the authors following [6]. The curves display good agreement, especially close to the maximum load values. Eight interface elements were used in this example with axes on the centroidal lines.

Table 1

<table>
<thead>
<tr>
<th>Properties for steel in Example 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Steel</strong></td>
</tr>
<tr>
<td>Beam</td>
</tr>
<tr>
<td>Reinforcement</td>
</tr>
</tbody>
</table>

Fig. 12. Example 3—load–displacement relation for 8, 12 and 16 element mesh.

Fig. 11. Example 3—constitutive relations.
6. Summary and conclusions

This paper presented the application of zero-thickness interface elements to the numerical modelling of composite beams with deformable shear connection. These types of structures are of common use in composite structures of steel and concrete, where the cost for the achievement of complete interaction is usually excessive. Their behaviours are characterized by relative axial displacements between the two constituent elements.

The use of interface elements provided a reasonable alternative to specifically developed beam finite elements with incorporated interlayer slip. Moreover, in situations where a number of beam layers may have relative slip the use of several interface element layers should be a very interesting modelling option.

The results obtained in the numerical examples are comparable in precision with the finite element solutions with equivalent interpolation schemes. However, more DOF must be employed. It has been shown that the proposed formulation also presents problems of slip locking when high values of connection stiffness are present. Nonetheless, it was shown that a translation of axes to the interface line alleviates the errors in the slip and curvature representations. With this procedure the actual slip becomes a linear function inside the element, and the eccentricity error does not cause error in the displacement evaluation.
Ongoing research will focus on the development of new interface elements which may enable the consideration of vertical uplift and time-dependent analysis.

Acknowledgements

The authors wish to thank USIMINAS and FAPEMIG for the financial support for this research. Support from CNPq, under Grant 479011/2006-0, is also gratefully acknowledged.

References