Influence of Bracings on the Stability of Columns

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Abstract: This article analyzes the stability of columns composed of deformable bars, with the particular goal of studying the influence of bracings. The bars are assumed to be deformable by bending, axial forces, shearing forces, and torsion. A computer program has been developed to determine the critical loading of the structure by taking into account geometric nonlinearity. The stiffness matrix technique is used to describe the structural response in three dimensions. Global instability is considered to have been achieved when a given loading introduces singularities in the global stiffness matrix. When the critical loading has been determined, it is possible to determine global instability parameters such as effective buckling coefficients. An example is presented and compared to ANSYS results to demonstrate the potential of the process.

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Introduction

Santos (2002) developed stiffness functions to analyze steel plane—framed structures consisting of deformable frames by bending, shear, and axial load. In these analyses, the effects of material nonlinearities in nonconservative systems were considered. According to the theory of small displacements, a computational program was developed to determine the global structural behavior. Using stiffness functions, this matrix method enables researchers to analyze the material nonlinearity of frames with little computational effort.

Mottram and Aberle (2002) used stiffness functions to show that the linear, nonlinear, and stability analyses of elastic structures with shear-flexible beam columns require a knowledge of the shear-flexible stability functions. These stiffness functions have been available in the literature since 1930. New functions were presented in which the shear flexibility was included in a linear matrix structural analysis format. Although these writers used a 6 x 6 elastic stiffness matrix, and our work uses a 12 x 12 elastic stiffness matrix, both studies incorporate the stiffness functions and the shear flexibility.

Equations for calculating bracing forces for individual members are contained in Appendix 6 of the AISC Specification for Structural Steel Buildings (AISC 2005). Fisher (2006) had an opinion that, since the top chords of trusses are designed using column strength equations, the column bracing equations are therefore appropriate for trusses.

A recent study on the use of bracing systems may hold great interest for engineers. Traditionally, steel bracing systems have been used to increase the lateral load resistance of steel structures. In recent years, the concept of steel bracing has also been applied to the retrofitting of reinforced concrete frames. Increased architectural flexibility, reduced structural weight, the ease and speed of construction, and the ability to choose more ductile systems have been considered to be the main advantages of steel bracing compared to RC shear walls. Maheri and Ghaffarzadeh (2008) studied how steel bracing systems for the seismic retrofitting of existing RC buildings, as well as for the seismic design of new buildings, revealing the importance of bracing systems studies to the study of structures in general.

This article considers three-dimensional (3D) structures with the particular goal of analyzing the influence of bracings on such structures. It also presents 3 stiffness functions that takes into account geometric nonlinearity.

This proposal is based on the following assumptions: (1) the 3D frame is composed of tubular bars that can be deformed by bending, axial forces, shearing forces, and torsion; (2) the bars are perfectly straight; (3) their cross sections have twofold symmetry; (4) their cross sections remain unchanged during the process; (5) the cross sections are idealized without warp; (6) the matrix development is accomplished based on the process of displacements for structures in the elastic range; (7) in the instability analysis performed on the 3D frame, the influence of the axial force over beam elements is taken into account, which is characterized by the geometric nonlinearity effect in the theory of small displacements; (8) for the sole purpose of starting the analysis, the axial forces P are obtained in the theory of first order; with the values of P the iterative process begins in the theory of second order; (9) the instability analysis is performed by considering the 3D structure, i.e., formed by the frame as well as by the bracing system; (10) the axial forces P are idealized without eccentricity; and (11) the loads are applied outside the frame plane as well as within the frame plane.

Based on these assumptions, the stiffness functions for the 3D...
Three-Dimensional Stiffness Functions

Stiffness Functions for the 3D Case

Table 1 shows a summary of stiffness coefficients. These expressions will be used to assemble the stiffness matrix of the 3D beam. Note that several of the factors can be changed according to the influence of the shear force or under axial force variation.

### Determination of 3D Frame Instability

Critical loading for the 3D frame is determined by the same technique used for the two-dimensional frame case described by Requena (1995, 1997) and Golub and Loan (1987). Instabilities in the global structure are found by checking the stiffness matrix for singularities as the vector of actions is gradually incremented. The stiffness function and finite-element methods have been compared by Callejas et al. (1998).

### 3D Frame Instability with Rigid and Elastic Connections

#### Elastic Connections

Elastic connections stand for intermediate constraints (somewhere between a null constraint and a full constraint). That is, such constraints might partially impair translations and rotations since there are no impediments in these coordinates. Thus, a 3D node could have as many as six elastic connections.
Elastic connections can be represented as elastic springs with their own stiffness constants. These constants can be incorporated into the global stiffness matrix as follows:

1. Identify the position of the coordinate affected by elastic constraint in the stiffness matrix.
2. Add the spring stiffness constant to the main through-brace of the global stiffness matrix, corresponding to the position of the elastic constraint.

The displacements obtained through such stiffness matrix take the elastic connection effect into consideration.

ANSYS

The ANSYS computer program (ANSYS 1995) used for result comparison uses the finite-element method. For this purpose each beam is discretized into several subelements. The elements used by this software are BEAM4 3-D and LINK8 3-D. The critical load is obtained using only elastic bending buckling, as ANSYS does not consider torsional buckling. This may explain the divergences found in some of the results.

Effect of Rigid and Elastic Connections

Bracings should not be considered structural elements as their constraints are different from those of the beams. That is, bracings are pin-connected to the frame. Furthermore, they demand a minimum stiffness to prevent displacements of the structure. A bracing’s constraints may vary throughout the structure’s lifespan, depending on the deformability of other bracings, the type of connecting between bracings, the type of connecting it to the other beams, the loading applied, and so forth.

In order to deal with such issues, more refined methods of modeling bracings have been intensively searched. According to Ballio and Mazzolani (1983), sophisticated tools were not available to calculate their influence on the structure. If they were, it would permit a complete global simulation of structure buckling. It was therefore very important for the engineer to apply common sense in addition to choosing appropriate approximation methods, many of which had already proven satisfactory and were common practices. The available tools allow professionals now to be more aware of the instability phenomenon and provide guidance on how to foresee problems.

Effect of Elastic Bracings on Structure Stiffness

We have developed a computer program to obtain the critical load $W_{crit}$, the buckling coefficients $K$, and the information required estimate the influence of bracings.

Buckling Coefficients

The computer program obtains $K$, given by Eq. (1), from the Euler formula (Thürlimann 1990):

$$ (KL)^2 = \frac{\pi^2 EI}{P_{crit}} \Rightarrow K^2 = \frac{\pi^2 EI}{P_{crit} L^2} \Rightarrow K_{(i)} = \sqrt{\frac{P_{Euler(i)}}{P_{crit}}} $$

The program’s version of $K$, denoted $K_{\text{prog}}$, is calculated separately for each beam. Its value is obtained from the critical load of the column; that is, the smaller value of the two sides in the practical case. Thus, in this program $K$ stands for the global behavior of the beam and the critical load is automatically defined.

Example of Braced Column

Assessing the actual influence of bracings is the main objective of this work to column bracings. Galambos and Xykis (1991) wrote a report about the problem of instability where the effect of bracings is considered.

The example was also simulated using the ANSYS program (ANSYS 1995). Each beam is divided into four elements, as the objective of this analysis is only to compare the deformed structures and not obtain the actual $W_{crit}$ value. For this reason, the $W_{crit}$ value reported by ANSYS is not accurate. In order to obtain an accurate $W_{crit}$ value, each element could be divided into at least 10 elements (Callejas et al. 1998).

Example: Columns

Steel columns are recommended to be braced along their weakest axis. In this manner the column’s buckling length is decreased while its critical load is increased. The goal of the example is to evaluate the influence of column bracings. Between frames, one bracing system is recommended to be placed every six spans. Each bracing system therefore typically supports three adjacent spans. This example is a set of five columns with one bracing system (Fig. 1).

As with the trusses, cross bracings are used in practical situations, even though they can only resist a tensile force. When the columns are loaded, only one of the bracings experiences a force. That is why just one set appears in the model. Again, in order to avoid transmission of bending moments to the columns, the bracings had their inertia reduced, so that this does not cause numerical problems in the program.

In practical terms, $W_{crit}$ of the column is usually calculated by adopting $K_y(\text{practical})=2$ for buckling around the $y$ axis and $K_z(\text{practical})=1$ for buckling around the $z$ axis. In addition, one uses angles for the bracing beams that are sufficient to guarantee column stability.

To check the accuracy of these practices, the bracing area was changed (using round and angle steel bracings) until we obtained the same $W_{crit}$ for the assembly as that of an isolated column, which is 4,638.0121 kN (for buckling around the $z$ axis). In this case, the profile of the bracings is not large enough to guarantee column stability. Table 2 shows the bracing areas, $W_{crit(\text{Assembly column})}$, $K_z$, and $K_y$, obtained by the program.

It is remarkable that $W_{crit(\text{Assembly column})}$ does not reach the $W_{crit}$ of an isolated column for any of the angle bracings employed. Not even a large area $A=100,000 \text{ mm}^2$ results in this critical value. We can conclude that the profiles themselves are not sufficient to guarantee the stability of the assembly; the buckling coefficients must be wrong. Hence, when employing angles for the bracing profiles, both $K$ values must be calculated.

According to the program, for $2 \times 2 \#1/4$ angle bracings, the values $K_y(\text{prog})=2.7414$ and $K_z(\text{prog})=1.2122$ are correct. This leads to the following critical loads:
\[ W_{\text{crit}}(y) = \frac{\pi^2(205)(73,275,000)}{((2.7414)(2,500))^2} = 3,156.3492 \text{ kN} \]

\[ W_{\text{crit}}(y) = \frac{\pi^2(205)(14,327,100)}{((1.2122)(2,500))^2} = 3,156.3368 \text{ kN} \]

Both values are slightly less than \( W_{\text{crit (Assembly column)}} = 3,156.3492 \text{ kN} \), so the practical value of 4,638.0121 kN, the isolated column used by designers may not be correct. For this application, the error decreases structural safety and should be more carefully analyzed. ANSYS (1995) arrives at \( W_{\text{crit (ANSYS)}} = 3,486.6736 \text{ kN} \), and presents the deformed shape displayed in Fig. 2.

If we greatly enhance the bracing area, for instance to 100,000 mm\(^2\), the deformed shape is shown in Fig. 3.

### Conclusions

1. For steel columns, which normally have heavier and quite rigid cross sections, it appears that equally rigid bracings are required. The example demonstrates that even when bracing cross sections are made unrealistically rigid, the expected theoretical values cannot be achieved. Therefore, commercial profiles should be adopted in these structures but their buckling coefficients should be determined by analyzing the global instability of the system. A detailed examination is needed to demonstrate the real structural behavior.

2. The ANSYS 1995 program does not provide accurate results when employed with small discretization, and permitting a large number of subdivisions in order to obtain more accurate results is sometimes impractical. No subdivisions are necessary for the program because the stiffness functions are used. The program presented here is thus practical for the daily applications of any engineer—it provides accurate results with little processing time.

3. All the analysis performed in this work made use of the elastic range hypothesis. In the forthcoming studies, this work will be extended to inelastic range.

4. The study of semirigid connections is also proposed.

5. Although the presented example is not fully conclusive, the potential and practicability of the program and the need for a global instability analysis for each structure have been amply demonstrated. Only after such analysis is it possible to guarantee real structural security for each case under analysis.

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References


Fig. 2. Deformed shape of example (angle steel bracing)

Fig. 3. Deformed shape of example (strong cross bracing)